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KOCHIN—LOITSYANSKII METHOD IN FREE CONVECTION PROBLEMS

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UDC 536.25

Freely convective heat transfer is computed by the Kochin—Loitsyanskii method on a vertical plate whose temperature is variable.

Integral methods used for approximate computations of freely convective heat transfer are based on the approximation of the exact velocity and temperature profiles in the boundary layer on polynomials or other functions (exponentials, for instance). The boundary-layer "thickness" and longitudinal velocity scale introduced provisionally are determined from the solution of the integral equations. A method using a particular class of exact solutions, a one-parameter family of self-similar profiles [1], exists in boundary-layer theory. A strictly defined quantity, the thickness of the momentum loss, which is a functional of the solution of the boundary-layer equations, is used as the scale of the transverse coordinate. We use this idea to compute freely convective heat transfer.

Let us consider free convection on a vertical plate with a given wall temperature ϑ_w. We assume that the energy dissipation and work of compression are negligibly small. Integral equations in a freely convective boundary layer have the form [2]

$$\frac{d}{dx}\int_0^\infty u^2 dy = g\beta\int_0^\infty \vartheta dy - \nu\left.\frac{\partial u}{\partial y}\right|_{y=0},$$
$$\frac{d}{dx}\int_0^\infty u\vartheta dy = -\frac{\nu}{\Pr}\left.\frac{\partial \vartheta}{\partial y}\right|_{y=0}. \quad (1)$$

Let us introduce the transformation scale

$$h(x) = \left(\int_0^\infty u\vartheta dy\right)^2 \Big/ \left(\vartheta_w^2\int_0^\infty u^2 dy\right),$$
$$U(x) = \vartheta_w\int_0^\infty u^2 dy \Big/ \int_0^\infty u\vartheta dy, \quad z = h^2/\nu \quad (2)$$

and substituting dimensionless functions in the equations, we obtain

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M. I. Kalinin Leningrad Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 4, pp. 724-727, October, 1980. Original article submitted September 24, 1979.

 0022-0841/80/3904-1140$07.50

TABLE 1. Results of Computations of the Self-Similar Problem

m	γ	f	F	G	Q	p
—0,6	1,0712	—1,4000	1,8667	0,4667	0,0001	2,2570
—0,55	0,9513	—1,0198	1,4371	0,4172	0,0810	1,7668
—0,5	0,8589	—0,7587	1,1381	0,3794	0,1327	1,4344
—0,45	0,7856	—0,5720	0,9216	0,3496	0,1668	1,1986
—0,4	0,7261	—0,4341	0,7597	0,3256	0,1899	1,0245
—0,35	0,6769	—0,3294	0,6352	0,3058	0,2058	0,8923
—0,3	0,6355	—0,2480	0,5373	0,2893	0,2169	0,7891
—0,2	0,5700	—0,1317	0,3951	0,2634	0,2304	0,6408
—0,1	0,5200	—0,0542	0,2981	0,2439	0,2371	0,5395
0	0,4809	0	0,2288	0,2288	0,2402	0,4676
0,2	0,4235	0,0690	0,1380	0,2070	0,2414	0,3730
0,4	0,3832	0,1097	0,0823	0,1919	0,2400	0,3146
0,6	0,3532	0,1357	0,0452	0,1810	0,2374	0,2752
0,8	0,3300	0,1535	0,0192	0,1726	0,2348	0,2473
1,0	0,3114	0,1661	0	0,1661	0,2324	0,2264
1,2	0,2961	0,1754	—0,0146	0,1608	0,2301	0,2103
1,4	0,2832	0,1825	—0,0261	0,1565	0,2280	0,1973
1,6	0,2723	0,1881	—0,0353	0,1528	0,2261	0,1871
2,0	0,2545	0,1961	—0,0490	0,1471	0,2229	0,1712
2,4	0,2406	0,2015	—0,0588	0,1427	0,2202	0,1597
3,0	0,2246	0,2069	—0,0690	0,1379	0,2170	0,1476
5,00	0,1920	0,2150	—0,0860	0,1290	0,2105	0,1264
10,0	0,1565	0,2202	—0,0991	0,1211	0,2056	0,1090

$$U\frac{dz}{dx}=F=-2\left[2f+p\int_0^\infty\theta d\eta+\frac{2}{\Pr}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}-\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0}\right],$$

$$z\frac{dU}{dx}=G=f+p\int_0^\infty\theta d\eta+\frac{1}{\Pr}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}-\left.\frac{\partial\varphi}{\partial\eta}\right|_{\eta=0},\tag{3}$$

where $f=(Uz/\vartheta_w)d\vartheta_w/dx$; $p=g\beta\vartheta_w z/U$.

We use a one-parameter family of self-similar solutions for a power-law change in the wall temperature [3] to approximate the velocity and temperature profiles. An exponent m in the law of surface temperature variation $\vartheta_w=Ax^m$ is the parameter. The scales U* and h* of the self-similar problem are known functions of x:

$$U^*=\frac{\nu}{x}(4\mathrm{Gr}_x)^{1/2},\quad h^*=x\left(\frac{\mathrm{Gr}_x}{4}\right)^{-1/4}.\tag{4}$$

Let us relate the parameter m in this family to the form factor f introduced. From (2) and (3) we find

$$U=U^*\varkappa(m),\ h=h^*\gamma(m),\ f(m)=4m\varkappa\gamma^2,\ p(m)=\gamma^2/\varkappa,$$
$$F(m)=2(1-m)\varkappa\gamma^2,\quad G(m)=2(1+m)\varkappa\gamma^2,\tag{5}$$

where

$$\varkappa(m)=\int_0^\infty\varphi^{*2}d\eta^*\Big/\int_0^\infty\varphi^*\theta^*d\eta^*;\quad \gamma(m)=\left(\int_0^\infty\varphi^*\theta^*d\eta^*\right)^2\Big/\int_0^\infty\varphi^{*2}d\eta^*.$$

The set of formulas obtained can be considered the parametric definition of U, h, p, G, and F as functions of f by using the parameter m.

Results of computations of the self-similar problem for Pr = 0.7 are represented in Table 1. We use it to eliminate the parameter m. Substituting the functions F(f) and G(f) determined from the solution of the self-similar problem into system (3), we obtain two ordinary differential equations. The solution of this system determines U(x), z(x), and f(x).

The dimensionless heat-elimination coefficient for the $Q(f)=-\gamma(\partial\theta^*/\partial\eta^*)_{\eta^*=0}$ known from the self-similar problem is given by the relationship

$$\frac{\mathrm{Nu}_x}{\mathrm{Gr}_x^{1/4}}=\left.\left(\sqrt[4]{\frac{x\cdot d\vartheta_w/dx}{p(f)f\vartheta_w}}\,Q(f)\right)\right|_{f=f(x)}.\tag{6}$$

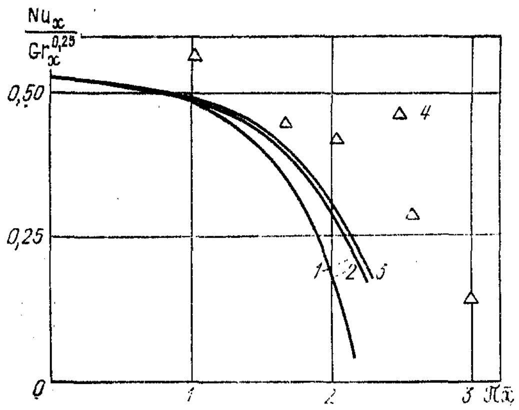

Fig. 1. Comparison of the results of computations and an experiment on the heat elimination on a vertical surface with a sinusoidal change in the wall temperature: 1) this method; 2) the local self-similarity method [5, 6]; 3) a numerical computation [7]; 4) experimental data [4].

The initial conditions for system (3) follow from the self-similar problem in the neighborhood of the leading edge with the parameter m_0, whose magnitude is determined from the condition

$$\lim_{x\to 0} \vartheta_w(x)/x^{m_0} = \text{const} \neq 0.$$

The most complete experimental and computational data on free convection in air on a plate with a sinusoidal change in the wall temperature $\vartheta_w = A \sin \pi \bar{x}$ [4-7] are available. In this case $m_0 = 1$. As follows from (4), (5), and (6), a passage to the limit is possible for $x \to 0$

$$\left.\frac{Nu_x}{Gr_x^{1/4}}\right|_{x=0} = \left(\frac{1}{\gamma\sqrt{2}} Q(f)\right)\Bigg|_{\substack{m=m_0\\ f=f(m_0)}} = \left(\frac{-1}{\sqrt{2}} \left.\frac{\partial\theta^*}{\partial\eta^*}\right|_{\eta^*=0}\right)\Bigg|_{m=m_0}. \tag{7}$$

Data obtained by the Kochin—Loitsyanskii method are compared with known numerical and experimental results on the local value of the heat-elimination coefficient in the figure. We see their good agreement. The local self-similarity method [5, 6] yields worse agreement with the numerical computation results, which is a consequence of the difference in the scale selection. In the local self-similarity method they are determined from the surface temperature distribution and are not related to the solution of the boundary-layer equations. As follows from (2), the scales h and U are defined as functionals of the solution of the boundary-layer equations, which implies the necessity to solve the ordinary differential equations (3). The existence of such "feedback" between the velocity and temperature distribution in the boundary layer and the scales is the advantage of the method proposed.

NOTATION

x, y, longitudinal and transverse coordinates; u, longitudinal velocity component; T, temperature; $\vartheta = T - T_\infty$, excess temperature; ν, coefficient of kinematic viscosity; a, thermal conductivity coefficient; g, acceleration due to gravity; β, coefficient of volume expansion; U, h, velocity and transverse coordinate scales; $\eta = y/h$, $\varphi = u/U$, $\theta = \vartheta/\vartheta_w$, dimensionless coordinate and functions; m, an exponent; $\varkappa$, γ, scale coefficients; $Pr = \nu/a$, Prandtl number; $Gr_x = g\beta\vartheta_w x^3/\nu^2$, Grashof number; $Nu_x = \alpha_x x/\lambda$, Nusselt number. Subscripts: w, magnitude on the wall; ∞, at a large distance from the wall; *, in the self-similar problem; 0, initial value or leading edge.

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MOTION OF A THERMAL WAVE FRONT IN A NONLINEAR MEDIUM WITH ABSORPTION

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UDC 536.24

We study the evolution of a thermal perturbation in a nonlinear medium whose thermal conductivity depends on the temperature and the temperature gradient according to a power law.

We consider an incompressible medium whose thermal conductivity depends on the temperature and temperature gradient according to the power law

$$k = k_0 u^\sigma |\mathrm{grad}\, u|^\alpha, \quad \sigma,\ \alpha = \mathrm{const} > 0.$$

Such a model of a medium, withgeneralization of the models used in the theory of nonlinear heat conduction [1], wasvalidated in [2] from the point of view of kinetic theory as a model of a medium with a finite relaxation time.

As follows from the results of [3], thermal perturbations in such a medium, unlike a medium with constant thermal conductivity, can be generalized to the form of thermal waves with finite velocity of displacement of the fronts. Below, we study the features of the motion of thermal wave fronts from an instantaneous point source of heat in the presence in a given nonlinear medium of volume absorption of thermal energy, the intensity of which depends on temperature according to a power law. Such absorption of thermal energy can be caused by processes of ionization and radiation in a high-temperature medium [1, 4].

The corresponding process of propagation of heat is described by a Cauchy problem for the quasilinear parabolic equation

$$\frac{\partial u}{\partial t} = \frac{a^2}{x^{s-1}} \frac{\partial}{\partial x}\left(x^{s-1} u^\sigma \left|\frac{\partial u}{\partial x}\right|^\alpha \frac{\partial u}{\partial x}\right) - \Pi u^\nu, \quad t > 0,\ x > 0,$$

$$u(0,\ x) = Q_0 \delta(x^s). \tag{1}$$

Here Q_0 is the energy of a point thermal source at the initial time; $\Pi = \mathrm{const} > 0$ is the coefficient of absorption, and s = 1, 2, and 3 for the cases of plane, axial, and central symmetries of the problem, respectively. Below, without loss of generality, we assume $a^2 = 1$, since by the choice of the time scale we can always reduce (1) to such form.

For certain values of the exponent ν in the lowest term of the equation we can find exact analytical solutions of the problem (1). An analysis of these solutions shows that

N. É. Bauman Moscow Higher Technical School. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 4, pp. 728-731, October, 1980. Original article submitted July 16, 1979.

0022-0841/80/3904-1143$07.50